Improved Meet-in-the-Middle Attacks on Reduced-Round DES

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Outline

Why Bother?

- We already know how to break the full DES!
- We have new, more powerful techniques which made MitM obsolete.
- We moved to AES!

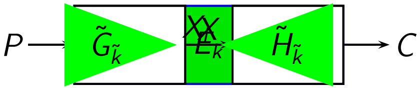
Motivation

- The retro movements hits Crypto!
  After seeing 2^{160} chosen plaintext attacks, trying to do stuff with small data complexity.
- Better understanding of some algebraic approaches (optimal sequence of guesses).
- DES-like structure are still in use (and promoted).

The Basics of Meet in the Middle (MitM) Attacks

- Consider a block cipher $E_k(\cdot)$ which can be written as $E_k(\cdot) = H_k(\cdot) \circ G_k(\cdot)$
- Let $C = E_k(P)$, and let $G_k(P) = X = H_k^{-1}(C)$
- Assume that a subset of bits i of X can be written as

$$X_i = \tilde{G}_{\tilde{k}}(P)$$
$$X_i = \tilde{H}_{\tilde{k}'}(C)$$



Performing a Meet in the Middle Attack

- **Identify** $\tilde{G}, \tilde{H}, i, \tilde{k}$, and $\tilde{k}'$
- Given a plaintext/ciphertext pair (P, C):
  1. For each $\tilde{k}$, compute $X_i = \tilde{G}_{\tilde{k}}(P)$
  2. For each $\tilde{k}'$, compute $X_i' = \tilde{H}_{\tilde{k}'}(C)$
  3. Only if $X_i = X_i'$ further analyze $\tilde{k}, \tilde{k}'$
- Further analyze may be: analyze another plaintext/ciphertext pair, exhaustively search remaining key bits, etc.

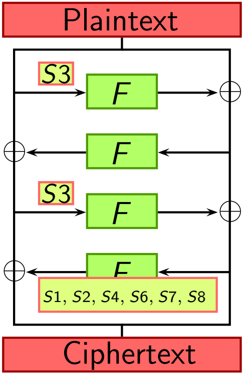
$$P \longrightarrow \boxed{\tilde{G}_{\tilde{k}}} \longrightarrow X_i \stackrel{?}{=} X_i' \longleftarrow \boxed{\tilde{H}_{\tilde{k}'}} \longleftarrow C$$

The Data Encryption Standard

- Proposed in mid'70 by IBM to NIST.
- Feistel block cipher with 16 rounds.
- Plaintext/ciphertext size — 64 bits.
- Key size — 56 bits.
- Each round function accepts 48-bit subkey.

Chaum & Evertse's Attack on DES

- Consider the first four rounds of DES.
- If the attacker knows the output from $S3$ in rounds 1 and 3, he can compute the MitM condition on 4 bits.
- The attacker guesses the subkeys of $R1/S3$ and $R3/S3$.
- One small problem though... **Guessing the key which enters $R3/S3$ is not sufficient, as the actual input itself is unknown.**
- The attacker has to guess the subkeys of $R4/S1$, $R4/S2$, $R4/S4$, $R4/S6$, $R4/S7$, and $R4/S8$.



Chaum-Evertse's Meet-in-the-Middle Attack on DES (II)

Due to the key schedule, $S1$, $S2$, $S3$, and $S4$ can be directly effected only by 28 key bits.

- $R1/S3$ and $R3/S3$ share 11 key bits.
- $R4/S1$, $R4/S2$, and $R4/S4$ introduce 8 more bits.

We guess 18 more key bits for $R4/S6$, $R4/S7$, and $S4/S8$, to obtain three of the bits which enter $R3/S3$.

The New Approach

Instead of guessing key, we guess internal state bits

- If for a key guess, there is no value of the internal state bits for which the MitM happens — the key is wrong.
- It might be the case that several internal state guesses remain for a given key guess.
- There is a tradeoff between the number of internal state bits which are guessed, and the probability that a wrong key is discarded.

An Attack Procedure Using One Known Plaintext

- For each guess of the 19 key bits,
  - For each guess of the 3 intermediate key bits, check the MitM on $R3/S3$. If no possible value, discard the key guess.
- Perform MitM on $R3/S2$ (guess 3 more key bits, and check for 4 more intermediate bits).
- Perform MitM on $R2/S1$ (guess 2 more key bits, and check for 4 more intermediate bits).
- ...
- After finishing the C register, there are about $2^{20.4}$ remaining values.
- Perform MitM on $R2/S8$ (guess 9 more key bits, check for 2 intermediate bits, *verify* two previously guessed intermediate bits).

An Attack Procedure Using Several Known Plaintexts

- It is possible to take several known plaintexts.
- If for any of the known plaintexts the key guess has no "corresponding intermediate bits", the key is wrong.

Guessed Intermediate Bits	Probability to "pass"
1	2^{-3}
2	$2^{-2.1}$
3	$2^{-1.3}$
4	$2^{-0.6}$
5	$2^{-0.2}$
6	$2^{-0.02}$

An Attack Procedure Using Chosen Plaintexts

- By using chosen plaintexts/ciphertexts, it is possible to fix the intermediate bits in all plaintext/ciphertext pairs to the *same* value.
- Thus, when a key "passes" the test with some intermediate value(s) for a given plaintext/ciphertext pair, it has to pass the test with *the same* intermediate value(s) for other plaintext/ciphertext pairs.
- This gives a much better filter for discarding wrong subkey guesses (and reduces time complexity significantly).

Chaum & Evertse's MitM Attack on 5-Round DES

- Guess 6 S-boxes in Round 1: $R1/S1$, $R1/S2$, $R1/S4$, $R1/S5$, $R1/S6$, $R1/S7$.
- Guess $R2/S3$.
- Guess $R4/S3$.
- Guess 6 S-boxes in Round 5: $R5/S1$, $R5/S2$, $R5/S4$, $R5/S5$, $R5/S6$, $R5/S7$.
- Perform MitM on 4 bits.

Number of guessed bits: 47.

Our MitM Attack on 5-Round DES

Observations:

- There are 24 bits used in $R1/S1$, $R1/S2$, $R1/S4$, $R2/S3$, $R4/S3$, $R5/S1$, $R5/S2$, $R5/S4$ — so it's better to guess these.
- There are 23 bits used in $R1/S5$, $R1/S6$, $R1/S7$, $R5/S5$, $R5/S6$, $R5/S7$ which determine only 6 intermediate bits.
- Guessing 6 intermediate bits has a very small chance of discarding wrong key guesses.
- We guess 8 bits more, and then we have to deal with only 4 intermediate bits (two from encryption side and two from decryption side).

Attacks on 4-Round DES

Attack	Data	Time
Differential	16 CP	Negligible†
Linear	52 KP	$> 2^{13.7}$ †
Algebraic [CB06]	1 KP	2^{46}
MitM [CH85]	1 KP	2^{35} †
MitM	1 KP	$2^{31.2}$
MitM	15 KP	$2^{20.0}$
MitM	6 CC	$2^{19.3}$

Attacks on 5-Round DES

Attack	Data	Time
Differential	64 CP	$> 2^{11.7}$ †
Linear	72 KP	$> 2^{13.8}$ †
Algebraic [CB06]	3 KP	$2^{54.3}$
MitM [CE85]	1 KP	$2^{45.5}$ †
MitM	51 KP	$2^{35.5}$
MitM	28 KP	$2^{37.9}$
MitM	8 CP	2^{30}

Attacks on 6-Round DES

Attack	Data	Time
Differential	256 CP	$2^{13.7}$
Linear	$>$ 104 KP	$2^{13.9}$ †
Algebraic [CB06]	N/A	$2^{50.1}$
MitM [CE85]	1 KP	$2^{52.9}$ †
MitM	1 KP	$2^{51.8}$

Conclusions

- There is a sequence of "good" guesses (which might explain the results of [CB06]).
- MitM might be useful on more rounds than previously believed.

Questions?

Thank you for your attention!