Dakota
– hashing from a combination of modular arithmetic and symmetric cryptography

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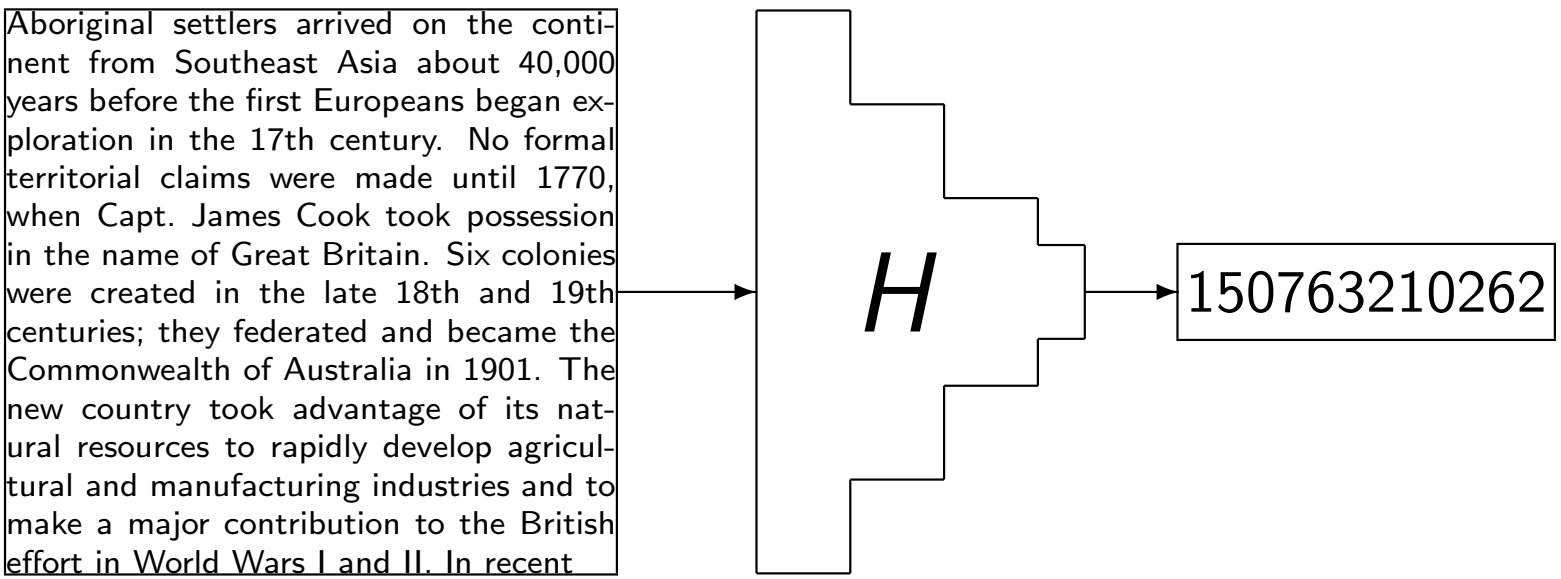
DaKoTa

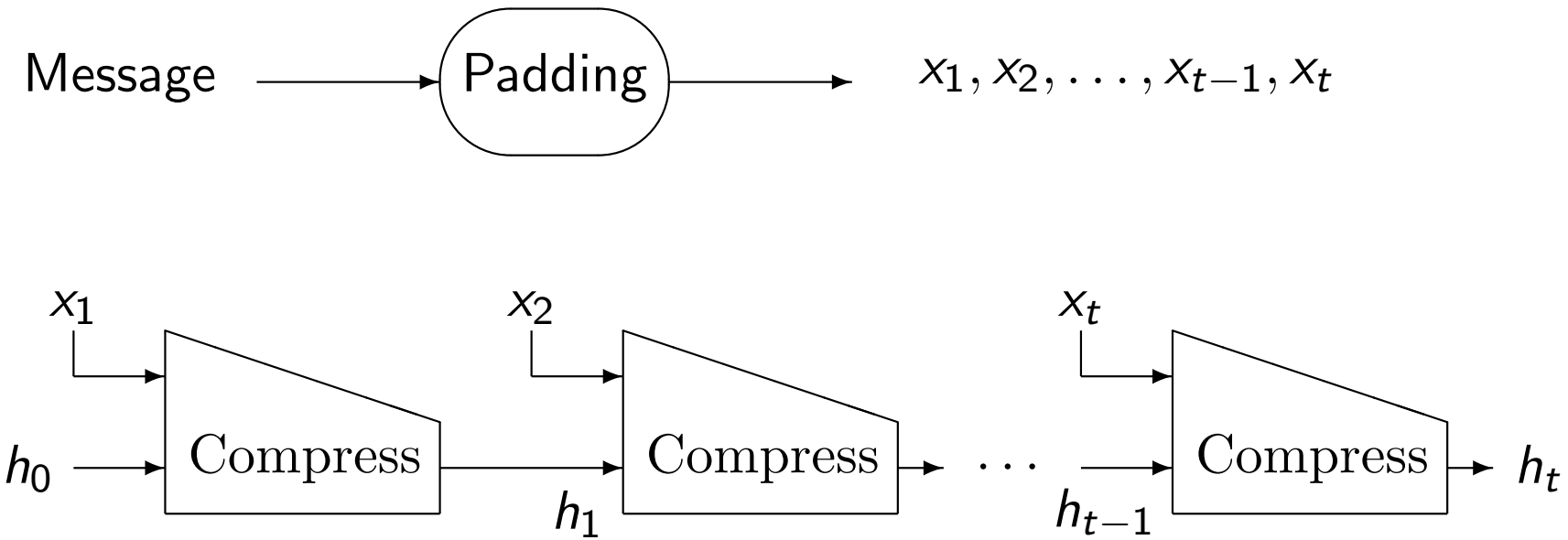
- DaKoTa, a hash function co-designed by
- Damgård, Ivan B.
- Knudsen, Lars R.
- Thomsen, Søren S.

Definition - hash function



$H : \{0,1\}^* \longrightarrow \{0,1\}^n$, for fixed value of n

Iterated hash functions



Damgård, Merkle, 89: collision for Hash $\Rightarrow$ collision for compress

History - brief

- 1980s: Hash functions based on block ciphers
- 1990s:
  - Dedicated, faster hash functions (kicked off by Rivest)
  - Many broken block cipher based hash functions
- 2000s:
  - Many dedicated schemes broken
  - Many new constructions, alternatives to Damgård-Merkle?
- Designs with proofs of security ?

Based on number theory

- Factoring
  - $n = pq$, $p \neq q$, large odd primes, α fixed, large order mod n.
  - $H(x) = \alpha^x \bmod n$
  - Collision: $H(x) = H(x') \Rightarrow x - x' = k\phi(n)$.
- Discrete log
  - Public primes: $p, q = \frac{p-1}{2}$, s.t. DLP(p) is hard
  - Public primitive elements of Z_p: α, β (randomly chosen)
  - $h(x, y) = \alpha^x \beta^y \bmod p$
  - Collision for $h \Rightarrow$ compute $\log_\alpha(\beta)$

"GMR"

- Based on GMR 1984
- $n = pq$, $p \neq q$, large primes, a_0, a_1 squares modulo n
- Public: n, a_0, a_1 $\qquad h : \{0,1\} \times \mathrm{SQ}(n) \to \mathrm{SQ}(n)$

$$h(b, y) \quad = \quad a_b \; y^2 \bmod n$$

- Collision gives "$\sqrt{a_0 a_1^{-1}}$" $\to$ factoring
- More efficient variants with more squares $a_0, \ldots, a_k$, Damgård 87

Number-theoretic hash functions

- most schemes slow, e.g., no real speed-up for use in digital signature schemes
- some schemes have unfortunate algebraic properties (may interact badly with other public-key algorithms)
- open problem to devise efficient "provably" secure hash function

Newer constructions

- VSH - Very Smooth Hash
  - Contini, Lenstra, Steinfeld, 2005
  - collision $\Rightarrow$ nontrivial modular square roots of very smooth numbers modulo N (composite)
  - efficient collision finder implies fast factoring algorithm
- LASH - A Lattice Based Hash Function
  - Bentahar, Page, Saarinen, Silverman, Smart 2006
  - based on the problem of finding small vectors in lattices

VSH - iterated hash function

- Let $N = pq$ be a public RSA modulus ($p \neq q$, both secret)
- Let $p_1, \ldots, p_k$ be public primes such that $\prod_{i=1}^{k} p_i < N$
  - Let $m = m_1, m_2, \ldots, m_{\ell k}$ be message, $m_i \in \{0, 1\}$
  - $x_0 = 1$
  - $x_1 = x_0^2 \ (p_1^{m_1} p_2^{m_2} \cdots p_k^{m_k}) \bmod N$
  - $x_{j+1} = x_j^2 \ \prod_{i=1}^{k} p_i^{m_{jk+i}} \bmod N$
  - $\text{Hash}(m) = x_\ell$

Getting to Dakota

- $n = pq$, $p \neq q$, large primes, $p \equiv q \equiv 3 \bmod 4$
- Public: n, a_0, a_1
- $h : \{0,1\} \times \mathrm{SQ}(n) \to \mathrm{SQ}(n)$

$$h(b, y) \quad = \quad a_b \; y^2 \bmod n$$

Getting to Dakota

- $n = pq$, $p \neq q$, large primes, $p \equiv q \equiv 3 \bmod 4$
- Public: n, f
- $h : \{0,1\} \times \mathrm{SQ}(n) \to \mathrm{SQ}(n)$
- $f : \{0,1\} \to \mathrm{SQ}(n)$

$$h(b, y) \quad = \quad f(b)\ y^2 \bmod n$$

Getting closer to Dakota

- $n = pq$, $p \neq q$, large primes, $p \equiv q \equiv 3 \bmod 4$
- Public: n, f
- $h : \{0,1\}^k \times \mathrm{SQ}(n) \to \mathrm{SQ}(n)$
- $f : \{0,1\}^k \to \mathrm{SQ}(n)$

$$h(x, y) \quad = \quad f(x)\ y^2 \bmod n$$

Getting even closer to DAKOTA

- $n = pq$, $p \neq q$, large primes, $p \equiv q \equiv 3 \bmod 4$
- Public: n, f
- $h : \{0,1\}^k \times \mathrm{SQ}(n) \to \mathrm{SQ}(n)$
- $f : \{0,1\}^k \to \mathbf{Z}_n$

$$h(x, y) \quad = \quad f(x)^2 \; y^2 \bmod n$$

Arriving at Dakota

- $n = pq$, $p \neq q$, large primes, $p \equiv q \equiv 3 \bmod 4$
- Public: n, f
- $h : \{0,1\}^k \times \mathrm{SQ}(n) \to \mathrm{SQ}(n)$
- $f : \{0,1\}^k \to \mathbf{Z}_n$

$$h(x, y) \quad = \quad (f(x)\ y)^2 \bmod n$$

Dakota- an iterated hash function

- $n = pq$, $p \equiv q \equiv 3 \bmod 4$, public: n, f
- $h : \{0,1\}^k \times \mathrm{SQ}(n) \to \mathrm{SQ}(n)$ $\qquad$ $f : \{0,1\}^k \to \mathbf{Z}_n$

$$h(x, y) \quad = \quad (f(x)\ y)^2 \bmod n$$

- Choose $r \in \mathbf{Z}_n^*$, let $s = r^2 \bmod n$
- Split padded message x into k-bit words, $x_1, \ldots, x_t$
- Set $y_0 = s$, then compute

$$y_i = h(x_i, y_{i-1}) = (f(x_i)y)^2 \bmod n$$

- Hash of x is then y_t.

DAKOTA- an iterated hash function $h(x, y) = (f(x)y)^2 \bmod n$

Assumption

Consider probabilistic polynomial time algorithm with input f, n, and output $x, \tilde{x}, z$. Probability is negligible that

$$x \neq \tilde{x} \quad \textit{and} \quad f(x)/f(\tilde{x}) = \pm z^2 \bmod n$$

Theorem

Hash function H is collision intractable under Assumption

find collision with prob $\epsilon \longrightarrow$ break Assumption with prob $\epsilon/2$.

Dakota- Assumption $h(x, y) = (f(x)y)^2 \bmod n$

Assumption

Consider probabilistic polynomial time algorithm with input f, n, and output $x, \tilde{x}, z$. Probability is negligible that

$$x \neq \tilde{x} \quad \textit{and} \quad f(x)/f(\tilde{x}) = \pm z^2 \bmod n$$

- f must be one-way: choose $z, \tilde{x}$, compute x
- f must be coll. resistant: find collision for f, let $z = 1$
- no circular argument?, since f does (need to) not compress
- $f(x) = x^2 \bmod n$ for $x > n/2$?? No! $x = z\tilde{x}$ breaks it
- $f(x) = x^3 \bmod n$
- $f(x) = x^2 + c \bmod n$ for $x > n/2$?? $f(x) = x^3 + c \bmod n$

DAKOTA- Proposal 1 for f $\quad h(x, y) = (f(x)y)^2 \bmod n$

$$f : \{0,1\}^k \to \mathbf{Z}_n$$

- Let n and n' be 1025-bit resp. 1024-bit RSA moduli
- Let $k = 1022$
- Let $u = x^2 \bmod n'$, where $x < n'/2$
- Let $v = E_{\kappa_1}(u) = v_1 \mid \cdots \mid v_8$, where E is CBC encryption using AES
- Let $f(x) = E_{\kappa_2}(v_8 \mid \cdots \mid v_1)$
- f is one-way, collision-resistant

Dakota- Proposal 2 for f $h(x, y) = (f(x)y)^2 \bmod n$

$$f : \{0,1\}^k \to \{0,1\}^k$$

- Let n be 1025-bit RSA-modulus, let $k = 1024$
- $f(x) = g(x) \oplus x$, where g is permuation of 1024 bits
- proposal for g:
  - tranform x into 8×8 matrix A with 16-bit values
  - Do 4 times
    $$A \leftarrow E(A)^T,$$
    where E is AES encryption (fixed key) of every column

Dakota- Performance $h(x, y) = (f(x)y)^2 \bmod n$

Hash function	Approximate speed (cycles/byte)
SHA-256	21.5
VSH	840
Dakota (Proposal 1)	400
Dakota (Proposal 2)	345

The end

Thank you